Proper Definition of Voltage for a Leaky Two-Layer Stripline Consistent with its Characteristic Impedance

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***Abstract* — Under suitable conditions, a non-uniform stripline having two different dielectric substrates on the two sides of its center conductor (referred to as a two-layer stripline), leaks power to its surrounding parallel-plate medium . The transverse fields of such a leaky line becomes strongly non-TEM in nature, with a non-conventional field behavior. This situation makes it impossible or ambiguous to define some of the basic, useful transmission-line parameters, such as voltage, transverse propagation power or a characteristic impedance. In this paper we investigate a suitable definition of the voltage of a two-layer leaky line, using which the characteristic impedance can be correctly computed. The practical validity of the definition, and its basic consistency with independent results are evaluated..**

I. Introduction

It is now well known that, under suitable physical conditions, printed transmission lines can leak power in the transverse directions. This occurs when the propagation constant of the transmission line is less than that of the background mode of the substrate structure [1-2]. Consider in particular a stripline structure having two different dielectric substrates on the two sides of its center conductor, as shown in Fig.1. We refer to this stripline configuration as a two-layer stripline. Power leakage can occur in this two-layer stripline, due to coupling to the fundamental parallel-plate mode of the two-layer structure, when the thinner substrate has a lower dielectric constant [2-3]. Unlike for a standard stripline with a uniform dielectric medium, the transverse fields of a leaky two-layer stripline are strongly non-TEM in nature. Therefore, the meaning of voltage becomes meaningless or ambiguous. The voltage computed by integrating the electric field across the center conductor and the top ground plane ($=V_2$) can become significantly different from that ($=V_1$) computed across the center conductor and the bottom ground plane. Accordingly, the characteristic impedance of the two-layer stripline, defined using a voltage-current method ($Z_c=V/I$), also become ambiguous, having significantly different values depending on what value of the voltage one uses. However, for a circuit modeling of the leaky transmission line it is important to define a suitable definition of voltage, which is consistent with its power flow, distributed circuit parameters, and characteristic impedance, as governed by a fundamental transmission-line theory. Independent methods of calculating the characteristic impedance, which are found to be consistent with the transverse power flow as well as with a distributed circuit model for the transmission line, have been presented in [4-6]. In this paper we investigate the meaning of voltage, and propose a definition which is consistent with the alternate established results of [4-6].

II. Theory

We first examine the total field of the transmission line. The total field can be decomposed into two distinct parts. The first part is the radiating leakage field, which grows exponentially in the transverse directions, and may be referred as the "growing field". This can be derived from the total field by properly extracting out the leaky-wave poles of the field expressions on the transverse-spectral plane. After the growing field is removed, the remaining field is tightly confined around the central conductor, and may be referred to as the "bound field". The growing field is produced as a result of radiation from the central strip into the parallel-plate medium, and constitutes the power loss from the central line. On the other hand, the bound field constitutes the guided-wave power along the transmission line [4,5]. Any definition of voltage should be consistent with the guided power along the transmission line. Therefore, it may be meaningful to define two voltages, computed by integrating the bound field across the top or the bottom substrate. Starting from the center conductor, the integration is performed along two different paths, one across the top substrate to the top ground plane, and the other across the bottom substrate to the bottom ground plane.

In addition to the above definition using the bound-mode field, we also try the conventional definition of voltage, which is obtained by integrating the total electric field between the center conductor and one of the ground planes. Like in the above case, two different values can be obtained by integrating across the two substrates. It will be interesting to study how the voltages obtained using the different procedures compare.

In summary, we examine in this study four independent values of voltages, computed across two different substrates, each using the total or the bound field of the transmission line - respectively without or with extracting out the poles on the transverse spectral plane. Once the voltages are defined, we compute a corresponding set of characteristic impedances, defined as Z_c=V/I, where the current is the independent variable, assumed to be unity. We compare these characteristic impedances with those computed independently using different methods [4-6].

We use a spectral-domain analysis employing spectral-domain Green's functions for the two-layer structure. The central strip is assumed to carry a known current having a uniform variation in the transverse direction, and a propagation factor $e^{-jk_e z}$ for variation along the propagation direction z. k_e is the complex propagation constant of the leaky line computed separately [2]. The spectral-domain method is particularly useful here. This allows us to conveniently separate the bound field from the total field by performing the spectral integration along the transverse spectral plane, with or without extracting the leaky-wave poles, respectively.

III. RESULTS

Fig.2 shows the phase and attenuation constants of a leaky two-layer line, as a function of the dielectric constant ε_{r2} of the thinner layer (top layer), plotted over a range $1<\varepsilon_{r2}<\varepsilon_{r1}=10.2$ where the line is found to be leaky. The line is seen to exhibit larger leakage (larger attenuation constant α) for a smaller value of ε_{r2}, as discussed earlier, with a limiting situation of no leakage when $\varepsilon_{r2}=\varepsilon_{r1}$.

We now compute the characteristic impedance of the geometry of Fig.2 using independent methods [4-6], that are known to be consistent with each other. Such studies of consistency between these methods have already been reported in [4-6], and is not duplicated here. We pick the "circuit-extraction method" of [6] (referred here as the "correct method") for comparison with the results obtained using the present voltage-current method. Figs.3(a,b) show the real and imaginary parts of the "correct Z_c," compared with two values obtained using the bound-mode fields. As can be seen, the result obtained using the bound-mode voltage across the top substrate (thinner of the two) closely compares with the correct Z_c, whereas that obtained using the bound-mode voltage across the bottom layer does not. The difference between the two values is more pronounced for lower value of ε_{r2}, when the leakage is stronger as seen from Fig.2. All three results converge to the same value at $\varepsilon_{r2}=\varepsilon_{r1}$, when the line turns into a purely TEM line with no leakage.

Fig.4(a,b) shows the real and imaginary parts of the characteristic impedance, for the same parameters of Fig.3, except the results are plotted as a function of the strip width W, with $\varepsilon_{r2}=5.0$. The comparison with the correct Z_c shows similar trends as in Fig.3. Characteristic impedance computed using the bound voltage across the top layer (thinner layer) is consistent with the correct value of the characteristic impedance, but deviates from that computed using voltage across the bottom layer (thicker layer). In this case, the deviation is larger for wider strip width W. For larger width W the transmission line "sees" the thinner substrate with a lower dielectric constant more than it "sees" the thicker substrate with a higher dielectric constant. This results in a progressively lower value of the phase constant of the line as W increases, compared to the propagation constant of the background mode which remain invariant with the strip width W. As per the leakage mechanism [2], this results in a larger leakage loss, and hence more pronounced non-TEM behavior, and thus greater deviation between the two sets of voltages.

We now compare the correct Z_c with those obtained using voltages across the two layers, employing the total electric field, instead of the bound field used in the computation of Figs.3,4. We particularly examine the imaginary parts in Figs.5(a,b), respectively as a function of ε_{r2} and W. We see that all three methods produce three distinctly different results. Also, the result obtained using the voltage across the thicker layer (bottom substrate) produces a positive value for the imaginary part of the characteristic impedance. This may be considered non physical. This is because a leaky stripline may be viewed as a transmission line having distributed loss resistances in series with the central conductor. From basic transmission line theory, such a situation can be shown to provide a negative imaginary part for the characteristic impedance [5].

III. CONCLUSION

The results of Figs.3,4 strongly suggest that the bound-mode voltage across the thinner layer, having a lower dielectric constant, is the preferred definition consistent with the correct characteristic impedance. Results of Fig.5 confirm that the total field of a leaky stripline does not describe the propagation power of the transmission line, and hence is not consistent with the correct characteristic impedance. This observation is also consistent with [4], which says a characteristic impedance computed using a power-current approach, where the power is computed using the bound-mode fields of the line, is the correct one.

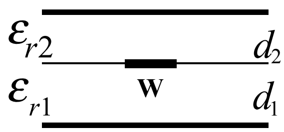

Fig. 1. The geometry of a non-uniform stripline, with two different dielectric substrates on the two sides of its central strip.

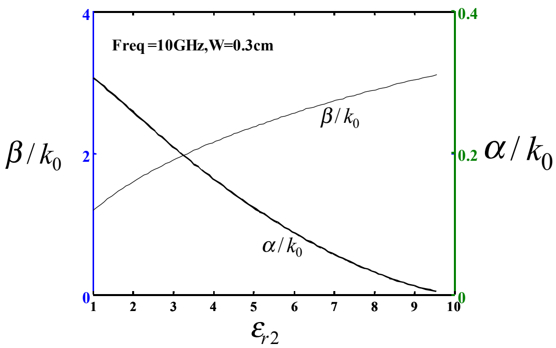

Fig. 2 The propagation constant β, and attenuation constant α, normalized to the free-space wave number k_0, for a leaky two-layer stripline. ε_{r1}=10.2, d_1=0.127cm, d_2=0.0254cm, W=0.3cm, frequency=10GHz.

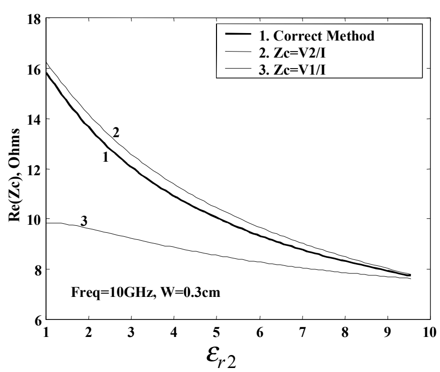

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Fig. 3(a) The real part of the characteristic impedance Z_c for the parameters of Fig.2, computed using different voltages, and compared with the correct value. V_2 and V_1 are the voltages computed across the layer above and below the strip, respectively, using the bounded part of the field.

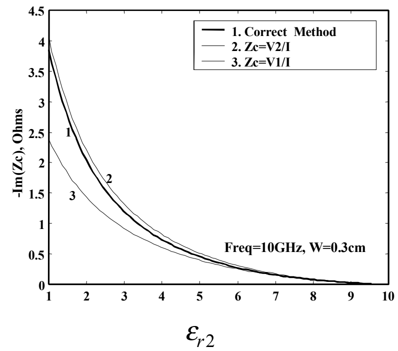

Fig. 3(b) The imaginary part of the characteristic impedance Z_c, for Fig.3(a).

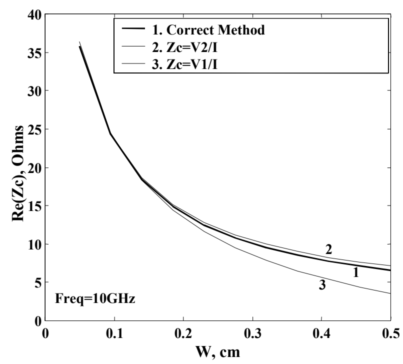

Fig. 4(a) The real part of the characteristic impedance Z_c, for a leaky two-layer stripline, with ε_{r2}=5, plotted as a function of the strip width W. Other parameters are the same as in Fig.3

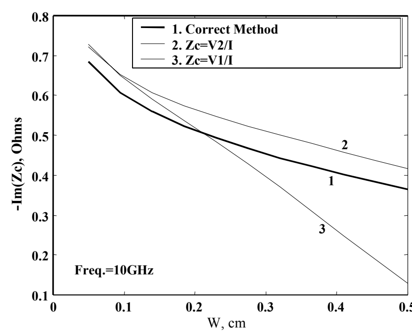

Fig. 4(b) The imaginary part of the characteristic impedance Z_c, for Fig.4(a).

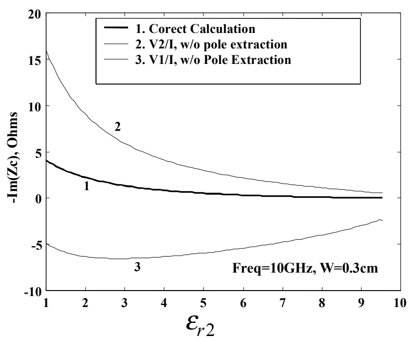

Fig. 5(a) Imaginary part of the characteristic impedance, for the parameters of Fig.3, but with voltages computed using the total field, which includes the bound as well as the leaky parts of the field. Mathematically, this is equivalent to not extracting out the leaky-wave pole in the spectral integral for computing the voltages.

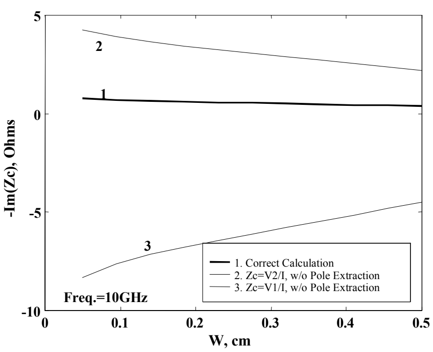

Fig. 5(b) Same as Fig.5(a), but plotted as a function of the strip width W, with ε_{r2}=5.0.

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